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Abstract

In scattering theory, the unitary limit is defined by an infinite scattering length and a zero effective range, corresponding to a phase-shift $\pi/2$, independent of energy. This condition is satisfied by a rank-1 separable potential $V(k, k') = -v(k)v(k')$ with $v^2(k) = (4\pi)^2(\Lambda^2 - k^2)^{-\frac{1}{2}}$, Λ being the cut-off in momentum space. Previous calculations using a Pauli-corrected ladder summation to calculate the energy of a zero temperature many body system of spin $\frac{1}{2}$ fermions with this interaction gave $\xi = 0.24$ (units of kinetic energy) *independent of density* and with $\Lambda \rightarrow \infty$. This value of ξ is appreciably smaller than the experimental and that obtained from other calculations, most notably from Monte Carlo, which in principle would be the most reliable. Our previous work did however also show a strong dependence on effective range r_0 (with $r_0 = 0$ at unitarity). With an increase to $r_0 = 1.0$ the energy varied from $\xi \sim 0.38$ at $k_f = 0.6 fm^{-1}$ to ~ 0.45 at $k_f = 1.8 fm^{-1}$ which is somewhat closer to the Monte-Carlo results. These previous calculations are here extended by including the effect of the previously neglected mean-field propagation, the dispersion correction. This is repulsive and found to increase drastically with decreasing effective range. It is large enough to suggest a revised value of $\xi \sim 0.4 \Leftrightarrow 0.5$ independent of r_0 . Off-shell effects are also investigated by introducing a rank-2 (phase-shift equivalent) separable potential. Effects of 10% or more in energy could be demonstrated for $r_0 > 0$. It is pointed out that a computational cut-off in momentum-space brings in another scale in the in principle scale-less unitary problem.

1 Introduction

The energy of a spin- $\frac{1}{2}$ fermion gas in the unitary limit is of current experimental and theoretical interest as evidenced by the many publications on this subject matter.

Our earlier results were presented in two previous papers [1, 2]. The total energy in units of the kinetic energy was found to be $\xi = 0.24$ independent of density. Other calculations as well as experiments show appreciably higher values, $\xi \sim .45$. [3, 4, 5, 6, 7, 8, 9, 10] This discrepancy is of considerable interest from a many-body theoretical point of view and is the motivation for this paper.

There are two distinctly different sources that determine the result of any many-body calculation. The first is the interaction the second the many-body theory used. If assuming a separable interaction the inverse scattering formalism allows for the construction of interactions (in principle an infinity number) that fit any given set of phase-shifts exactly. If the phases do not change sign (and no coupling between states exist) a rank-1 potential is sufficient for a complete numerical fit. In the unitary limit, defined by an infinite scattering length, $a_s \rightarrow \infty$, and effective range $r_0 = 0$, the rank-one separable interaction $\langle k|V|k' \rangle = v(k)v(k')$ fitted to phase-shifts $\delta(k) = \frac{\pi}{2}$ for $k \leq \Lambda$ is given by [1]:

$$v^2(k) = \frac{(4\pi)^2}{(\Lambda^2 - k^2)^{\frac{1}{2}}} \quad (1)$$

In the limit $\Lambda \rightarrow \infty$ this interaction reduces to a delta-function in coordinate space. Any numerical calculation necessitates a cut-off in momentum-space and as shown by eq. (1), this requires the strength to be increased with increasing k and a singularity at $k = \Lambda$, that in general makes this interaction difficult to handle numerically. An analytic result for the total energy could however be obtained in a specified approximation [1] yielding the value quoted above, $\xi = 0.24$, independent of density. A strong dependence on the effective range was however also reported. It was for example found that if increasing $r_0 \rightarrow 1 fm$, ξ increases to ~ 0.4 close to other reports (see citations above).

Calculations, other than the 'exact' at $r_0 = 0$ were found to be computationally difficult for $r_0 < 0.25$. This situation can be understood by studying Fig. 1, showing a big difference in phase-shifts between the two cases $r_0 = 0$ and $0.25 fm$ respectively. This figure also shows that the corresponding potentials are dramatically different because of the singularity at $k = \Lambda = 8 fm$ in the unitary limit.

Any fit to the on-shell phase-shifts alone does of course not define the interaction uniquely except exactly at the unitary limit as argued below. But one advantage of the inverse scattering method is that off-shell

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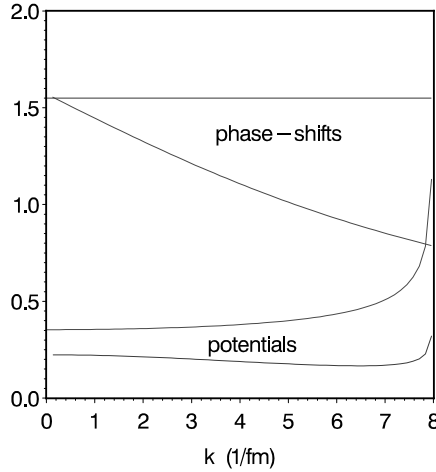


Figure 1: There are two sets of curves. The upper are phase-shifts the lower the corresponding rank-1 potentials defined by $v(k)$. The uppermost in each set is for the unitary case, $a_s \rightarrow \infty$ and $r_0 = 0$, the lower in each set is for the 'near' unitary case with $r_0 = 0.25$. Unit is Fermis ('fm').

properties can easily be monitored and changed by increasing the rank of the potential while maintaining the on-shell fit to phase-shifts. Results of such a modification of the interaction will be shown below.

The most accurate many-body theoretical method is at least in principle the Monte Carlo, although it may be hampered by computer-technical limitations. The Brueckner nuclear many-body theory was the first to (with considerable success) describe nuclear matter saturation and finite nucleus properties. This theory defines an effective in-medium interaction which can be regarded as a modification of the free scattering T -matrix:

$$T = v + v \frac{1}{e_0 + i\epsilon} T \quad (2)$$

or more relevant the reactance matrix \mathcal{K} :

$$\mathcal{K} = v + v \frac{\mathcal{P}}{e_0} \mathcal{K} \quad (3)$$

where \mathcal{P} indicates the principal value and e_0 kinetic energies. These free scattering interactions are modified by the definition of an in-medium effective interaction, the Brueckner Reaction Matrix involving two important modifications that can be physically justified. The first recognizes the statistics of a Fermion gas and the resulting Pauli-blocking in the many-body system. The second modification was in accordance with a, at the time of incipience of this theory, new idea, the mean field concept, a corner stone in the theory of both the shell and the optical model. These two effects are included self-consistently in the Brueckner Reaction Matrix K defined by:

$$K = v + v \frac{Q}{e} K \quad (4)$$

where $e = e_0 + U$. The particle propagator now includes interactions with neighboring nucleons via the mean field U defined self-consistently by the in-medium K -interaction. As further detailed in Sect. (2) this is in effect a 3-body term. The Brueckner theory as defined by the effective interaction (4) implies a summation of a particular sub-set of perturbation terms (diagrams). Considerable effort has in the past been made to estimate corrections to this but they were all relatively small (or too uncertain to be regarded) and rarely included in what is commonly referred to as "Brueckner Theory". This theory was primarily designed to be applied to nuclei. There is no à priori reason why it should be applicable to the problem at hand with a substantially

different interaction. (Even though neutron matter sometimes is quoted as being 'close' to a unitary gas, our results suggest it to be a poor approximation.) Although the estimates of higher order terms in nuclear matter calculations show them to be relatively small, these terms may be important in the unitary system because of the rather different interaction. This is however a question beyond the scope of this investigation.

From a many-body theoretical point of view it is rather of interest to find if Brueckner theory as formulated above will agree with Monte Carlo and other reported results for the energy of a unitary gas.

Our previous calculations with the rank-1 potential at or near the unitary limit referred to above were made using Brueckner theory, but neglecting the mean-field U above, i.e. with $e \equiv e_0$.

The effect of including the mean field in hole- (but not particle-) line propagation is now also investigated with results shown below in Sect. 2. Sect. 3 deals with effects of off-shell modifications, while Sect. 4 contains a summary and some conclusions.

2 Mean-field effects

In the many-body theory of nuclei the effect of nucleons propagating not as free but rather as bound in a shell-model potential (the mean field) is extremely important and is a major contributor to nuclear saturation and finite nucleus stability against collapse. This effect is included in the Brueckner K -matrix by using $e = e_0 + U$ rather than just e_0 (kinetic energy) in eq. (4) and is in the literature referred to as a *dispersion* correction. It can be estimated as follows. Let ΔU be the averaged difference between hole and particle potential energies. The dispersion correction ΔK_{disp} is then found by differentiating K in eq. (4) by the energy denominator e (or U) to get [13, 14]

$$\Delta K_{disp} \propto \Delta U * I_w. \quad (5)$$

where the wound-integral I_w is defined by

$$I_w = \int (\Psi(r) - \Phi(r))^2 d\mathbf{r}$$

with Ψ and Φ the **correlated** and **uncorrelated** two-body wave-functions respectively. It is relatively small at low density (small finite nuclei) but grows with density because of the increased binding. And it is repulsive which explains why it is an important contribution to saturation. It is basically a three-body effect as the effective two-body interaction depends on the presence of "third nucleons" that constitute the mean field U . And it is important that in nuclei, U is momentum-dependent (non-local) so that $\Delta U \neq 0$. The wound-integral I_w is an important quantity by itself being a measure of the correlation strength.

Calculations have shown that in nuclei this dispersion effect is less important for the 1S_0 interactions but more so for the $^3S_1 - ^3D_1$ interactions. Eq (5) shows it to be proportional to both ΔU and I_w . The 1S_0 interaction is defined (partially) by a scattering length and an effective range $a_s = -18.5 fm$ and $r_0 = 2.8 fm$. The reason the dispersion effect is small for this state in nuclei is that I_w is small for this interaction. It is appreciably larger in the coupled $^3S_1 - ^3D_1$ state. It is found below that approaching the unitary limit with $a_s \rightarrow \infty$ and $r_0 \rightarrow 0$, I_w will increase. But ΔU will decrease because the mean-field becomes less momentum-dependent so that the difference between the hole and the particle potential energies becomes small. The latter effect motivated the neglect of the dispersion effect in the Brueckner-equation in our previous unitary calculations.

In the results presented in this report the mean field *is* included, but *only for hole-propagation*. The justification for this choice goes as follows. The difference between mean-field interactions of nucleons propagating as holes or as particles respectively was shown already by Brueckner and Gammel [15, 11] and further investigated in many papers e.g. [16]. In most later nuclear calculations one has however, without strong mathematical proof, reverted to treating the hole and particle propagations on an equal footing. (The "continuous" choice). As emphasized by Bethe[17, 12] these insertions are basically 3-body interactions and can be treated as such by the Faddeev method as he also did.

The mean field in the nuclear medium has a definite momentum-dependence with $m^* \sim 0.8$. This is not the case in a gas in the unitary limit. Although important in nuclear many body theory there is therefore no *a priori* reason why the inclusion of the hole-line interactions alone should provide a better approximation for the energy of the unitary gas. It seems however worth-while to investigate this effect in more detail.

Some results of our calculations are shown in Fig 2. The lowest curve is for free propagation without a mean field ($U \equiv 0$) at $k_f = 1 fm^{-1}$. (Fig. 3 in ref. [2] shows results also at other densities with $U \equiv 0$.) Note that the

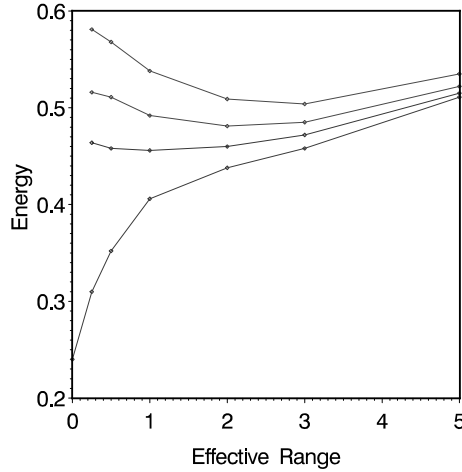


Figure 2: Energy, in units of kinetic energy, is shown as a function of effective range r_0 (in units of fm). Scattering length is $a_s \rightarrow \infty$. The lowest curve shows the energy calculated without any insertion in hole lines. The remaining three curves show, from top to bottom, the results with insertions being $1.0, \frac{1}{2}$, and $\frac{1}{4} \times U_h(k)$ respectively, where U_h is the selfconsistently calculated mean field for hole lines, i.e. for $k \leq k_f$. The fermimomentum is $k_f = 1 fm^{-1}$ as in all calculations in this work.

point at $r_0 = 0$, the unitary limit, taken from the previous (mostly) analytic calculation in ref. [1] is reached by smooth extrapolation from the data obtained in the present calculations. The upper three curves show that the effect of hole-line insertions, the dispersion correction, as mentioned above is repulsive. One furthermore sees that this repulsion *increases* with *decreasing* effective range. In accordance with eq. (5) this is associated with an increase in the wound-integral I_w , i.e. correlations, with decreasing effective range. The mean field U_h is on the other hand found to be essentially independent of r_0 . The important message of the results shown here is the rather dramatic change in the r_0 dependence of ξ nearly nullifying it for $U(k) = \frac{1}{4}U_h(k)$. Further comments on these results are found in Sect. 4.

The results shown in Fig. 2 (as in most results reported here), were made with a cut-off in momentum space $\Lambda = 10 fm^{-1}$. Fig. 3, on the other hand shows the near independence of Λ . The middle curve, in particular, shows this to be so for $\Lambda > 2k_f$, i.e. the 'range' of the Q - (Pauli-)operator. There are no insertions ($U \equiv 0$) in the propagator lines in this curve. The upper curve does include the repulsive effect of hole-line insertions. It has a slight bump at smaller cut-offs. A very similar result was shown in a previous work ([14]) for nuclear forces. The unitary limit is unique. The only energy-scale is the kinetic energy. This was explicitly shown to be the case in the ladder-approximation with the interaction of eq. (1) where the quantity ξ was shown to be independent of density as is expected in the unitary limit. [1] In that calculation the limit $\Lambda \rightarrow \infty$ could be reached explicitly and exactly which made this calculation unique. Fig. 3 in ref. [2] on the other hand, shows a definite density-dependence when $r_0 \neq 0$, even though $a_s \rightarrow \infty$.

A finite value of Λ does however bring in another scale in these calculations. It is evident that for any fixed value of Λ a density-dependence has to exist, because for $k_f > \Lambda/2$ (i.e. $\Lambda < 2k_f$) the solution of the reaction-matrix K would involve momenta that are larger than those restricted by Λ . This explains the rapid increase in energy for $\Lambda \leq 2 - 3 fm^{-1}$ shown in Fig. 3. Only for appreciably larger values of Λ does one see a constancy at least in case of $r_0 = 1 fm$. The smallest value of r_0 for which reliable calculations could be made here was $r_0 = 0.25 fm$. The Λ -dependence for this value of r_0 is shown by the lowest curve in Fig. 3. It does indeed show a larger but maybe not significant increase in Λ -dependence.

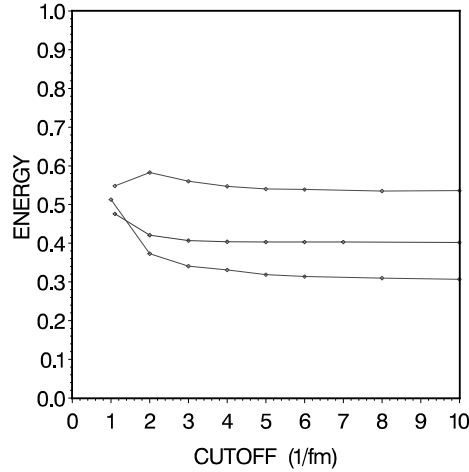


Figure 3: The middle curve shows the energy, in units of kinetic energy without insertions in hole lines, i.e. only kinetic energy in the propagators. The upper curve includes the self-consistent mean field U_h (see text). The horizontal axis is labelled with the cutoff Λ in units of fm^{-1} . The interaction is for these two curves defined by $r_0 = 1fm^{-1}$ and $a_s \rightarrow \infty$ and a rank-1 potential. The lowest curve is similar to the middle except that the effective range now is $r_0 = 0.25fm$, i.e. somewhat closer to the unitary limit for which $r_0 = 0$.

3 Off-shell scattering

A potential interaction fitted to scattering phase-shifts by inverse scattering or any other method, is not unique. This is of course a reason why so much effort has been expended to construct meson-theoretical and QCD derived NN-interactions for use in many-body calculations. The phase-shifts are on-shell data. The inverse scattering method is based on inverting the reactance-matrix \mathcal{K} of eq. (3) with the diagonal elements given by $\langle k|\mathcal{K}|k \rangle = \tan \delta(k)$. In any theory that has the purpose of explaining the properties of a many-body system from two-body data this on-shell diagonal information would be necessary but in general not sufficient. In any many-body theory and specifically in the case at hand, Brueckner theory, off-diagonal elements are needed to construct the in-medium off-shell interaction, the reaction matrix defined by eq.(4). No direct experimental off-shell data are however in general available for this purpose as it is not an observable[18] and in fact indistinguishable from many body forces. Their relative contributions ('strengths') are indistinguishable and not subject to observation referred to as 'the equivalence theorem'.

Having said that, the problem still remains: To what extent, numerically does the result of a many-body calculation depend on variations in off-shell propagation. Such a study is very conveniently done within the inverse scattering formalism. To demonstrate this, consider the phase-shifts defined by some scattering length and effective range. If they are all of the same sign they can (easily) be reproduced by a rank-1 separable potential. The low-energy ($E < 150MeV$) nuclear 1S_0 interaction is a good example and a rank-1 potential is in this case a "good" representation. This is exemplified by the close numerical agreement with the Bonn off-diagonal elements shown in Fig. 3 of ref. [20]. It can be understood to be a consequence of the appearance of a pole in the scattering matrix near the real axis which is the case for potentials with large scattering length and small effective range. For other states, in particular the 3P_1 a rank-1 potential is sufficient to fit the phase-shifts but it disagrees with results of the Bonn-potentials and this can be traced to a difference in off-diagonal reactance matrix elements. [20]

Off-diagonal elements can in fact easily be changed, while preserving the on-shell phase-shift information, by increasing the rank. It was shown already by Chadan[21] and by Fiedeldej[22] how to do this. The method assumes a given set of phase-shifts $\delta(k)$ and an arbitrarily chosen potential with some associated phase-shifts

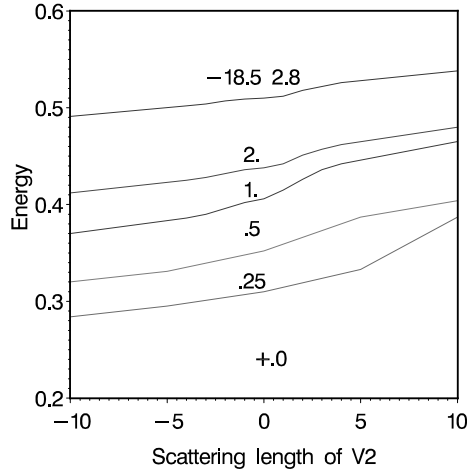


Figure 4: The energy, in units of the kinetic energy, is shown as a function of the scattering length a_s of potential V_2 of a rank-2 potential as described in the text. The phase-shifts $\delta_0(k)$ are defined by this value of a_s while $r_0 = \pm 10$ with the sign opposite to that of a_s . The 'given set of phase-shifts', $\delta(k)$ is in case of the top curve defined by a_s and r_0 as indicated, while for the lower curves only r_0 is indicated while $a_s \rightarrow \infty$. The cross indicates the previously reported result in the unitary limit $r_0 = 0$. [2] Unit is Fermis (fm).

$\delta_0(k)$. A second potential can then be calculated so that the sum, a rank-2 potential reproduces the given set $\delta(k)$. This method was used by Chadan to show the effect of varying off-shell behavior. (The method was extended by Fuda[23] to coupled channels and used in a previous work by Kwong and the author [20]).

This method to study the effect of off-shell variation is also used here and results are shown in Fig. 4. In the uppermost curve the given set of phase-shifts was defined by the nuclear 1S_0 scattering length $a_s = -18.5 fm$ and effective range $r_0 = 2.8 fm$, while the four lower curves show results for $a_s \rightarrow \infty$ with $r_0 = 2, 1, .5$ and $.25 fm$ as indicated in the diagram. The second part of the rank-2 potential V_2 , 'arbitrarily chosen', was defined by $r_0 = \pm 10$ while a_s is given by the numbers along the horizontal axis, i.e a potential of relatively short range in momentum-space. We wish to emphasize that every point along each of these curve are equivalent in the sense that the potential at each point all have the same scattering same phase-shift. They are phase-shift equivalent. Note that $V_2 \equiv 0$ when the scattering length $a_s = 0$, and the slope is a measure of an 'off-shell dependence'. Regarding for example the $r_0 = 0.25$ curve ($a_s \rightarrow \infty$) one finds $0.28 < \xi < 0.38$; there is no unique answer to what the energy is here, if phase-shifts are the only information used to define the interaction.

The diagram illustrates the point that the energy (and other properties) of a many-body system is not defined by on-shell properties alone; the potential is not defined solely by the phase-shifts. It is well-known that the off-shell effect is equivalent to (indistinguishable from) that of many-body forces so that the curves could also be interpreted as being functions of the strength of some (unspecified) 3-body force. It is however important to realize that there is no exactly solvable N-body theory for $N \geq 3$. Higher order terms could also change with off-shell. There might even be cancellations.

In contrast, the three-body problem e.g. the triton, is exactly solvable and off-shell (three-body) effects can be calculated (see for example ref. [19].)

The results shown in Fig. 4 are for $r_0 \geq 0.25$. For smaller values the calculations become inaccurate related to the singular nature of the interaction in the unitary limit as shown in eq. (1) and illustrated in Fig. 1. In this limit the main part of the calculation can however be done analytically if $V_2 = 0$ and the result is $\xi = 0.24$ as reported in ref[2] and shown by the cross in Fig. 4.

The results shown in Fig. 4 show a definite off-shell dependence even for the smallest value of r_0 . In the unitary limit $r_0 \rightarrow 0$ such a dependence should not exist in an exact calculation but may of course still be

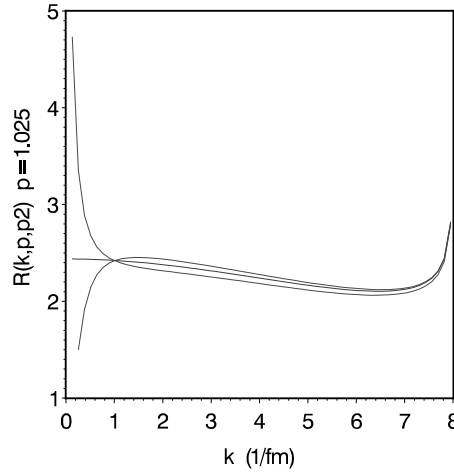


Figure 5: Half off-shell elements of the reactance matrix (eq. (3)) for three phase-shift equivalent potentials as detailed in the text. Unit is Fermis ('fm').

seen in the ladder-approximation used here. unitary interactions used in at least most calculations are only approximate in that the limit $r_0 = 0$ is not satisfied. One purpose of the present work is to exemplify that this may lead to uncertainties e.g. in the determination of ξ also due to unknown and unspecified off-shell properties.

For further illustration Fig. 5 shows half-shell reactance matrix elements. (See eq. (3)). The middle curve is obtained with a rank-1 potential V_1 fitted to phase-shifts defined by a scattering length $a_s \rightarrow \infty$ and an effective range $r_0 = 0.25$ while $V_2 \equiv 0$, i.e. the same parameters as for the lowest curve in Fig. 4 at the abscissa point $=0$. The upper and lower curves are with rank-2 potentials with the same parameters as at the endpoints of that same curve in Fig. 4. The three curves differ in particular at small momenta, a consequence of the short range (in momentum-space) of V_2 . The three curves intersect at the diagonal point $k = p = 1.025 \text{ fm}^{-1}$ because the three different potentials all fit the same phase-shifts and the diagonal of the reactance matrix is $\mathcal{K} = \tan(\delta)/k$. Note the rise of the curves as $k \rightarrow \Lambda = 8$. This is consistent with the effective range $r_0 = 0.25$ being fairly close to the unitary limit $r_0 = 0$ whence the potential is singular at $k = \Lambda$ as shown by eq. (1). (See also Fig. 1).

4 Summary and Conclusions

Our previously reported ladder calculation of the total energy of a unitary gas [2] gave the result $\xi = 0.24$ (energy in units of kinetic energy of the uncorrelated gas). The unitary interaction (1) used in that calculation was obtained by inverse scattering and it satisfies the unitary condition exactly by having scattering phase-shifts $\delta(k) = \pi/2$ for all $k < \Lambda$, the momentum cut-off. Although not claimed to be an 'exact' many-body calculation it had the virtue of being an almost completely analytic result. It also relied to some extent on the successful Brueckner approach to the nuclear many-body problem, being a ladder-summation and respecting the Fermi-statistics of the problem. It did however neglect another aspect of the Brueckner method, the mean-field (self-energy) insertions in propagator-lines.

But an important result of the previous calculations was also that the energy increases rapidly with increasing value of r_0 , while keeping the scattering length large. This is also shown here by the lowest curve in Fig. 2 above.

In the present report two separate issues were addressed. One is the non-uniqueness of interactions derived from the on-shell scattering phaseshifts. The other is the previously neglected insertions in hole-lines.

Regarding the first issue it is basically unknown territory. The 'potential' is not an observable, but a

theoretical construction. This is for example illustrated by the nucleon-nucleon 1S_0 -potentials shown in Fig. (3) of ref. [24] all fitting low-energy phase-shifts but differ with Λ . In the present calculations the interactions are assumed to be purely 2-body. In the real world, e.g. some atomic gas, it may be necessary to introduce higher order forces to allow for 'distortions' of the force-field due to neighboring particles. The results shown in Fig. 4 illustrate the fact that phase-shifts alone do not define the properties of a many-body system. Fit to phase-shifts may be considered a necessary condition in this type of many-body theory. But it is not a sufficient condition to predict the properties of a many-body system. The calculations presented here fulfill the first, the necessary condition but considerable variations in energy can be seen in Fig. 4 if off-shell properties are changed. The unsolved problem is of course how to relate these to specific gases. We may however also argue that in the unitary limit the rank-1 potential (1) fulfills both conditions, the correct on- as well as off-shell dependence. As referred to above, it has already been demonstrated that the nuclear 1S_0 interaction with considerable success can be represented by a separable potential. This could be attributed to a nearly bound state in this case. In the unitary limit the pole of the scattering matrix lies not only close to but exactly on the real axis. The interaction should therefore in this case be an even better candidate for being represented by a separable potential at least in the vicinity of this pole. (See e.g. ref. [25] regarding this subject). Question is, is the separable interaction given by eq. (1) unique. It certainly is if one restricts to small momenta.

As to the mean-field propagation in the definition of the effective in-medium interaction, it is of utmost importance in the nuclear many-body system and it is included in the definition of the Brueckner reaction matrix, eq. (4). It is the main reason for nuclear saturation in Brueckner theory of nuclei, and that is mainly a consequence of the momentum-dependent mean field $U(k)$ so that $\Delta U \neq 0$. The result of this is that the dispersion correction, eq. (5) is large and increasing with density.

The situation is different for the case studied here. The mean field $U(k)$ is nearly constant for the unitary short-ranged interaction. The mean field is practically independent of momentum so that ΔU in eq. (5) would be nearly zero with a *continuous* choice. There are however other facts to consider here. The first is that I_w in eq. (5) increases as r_0 decreases approaching the unitary limit. Referring to works by Brueckner, Gammell, Bethe and others it was pointed out in Sect. 2 that the determination of ΔU involves an effort to sum higher order terms in the calculation of the energy. The mean-field $U_h(k)$ for $k < k_f$, to be used for insertions in hole-lines should be calculated from K as defined by eq. (4). Regarding $U_p(k)$ with $k > k_f$ the situation is not that clear. Bethe [17] and others did for example suggest to choose $U_p(k) = 0$, in the literature often referred to as the *standard* choice.

The effect of hole- and particle- insertions was investigated above. Referring to Fig. 2 one notes that without any insertions the energy decreases rapidly as r_0 is decreased agreeing with an earlier report.[2] One finds on the other hand a dramatic opposite effect with the standard choice. By reducing the mean field by a factor of $\frac{1}{4}$ one sees that $\xi = 0.4 \Leftrightarrow 0.5$ and almost independent of r_0 .

These results do not conclusively suggest a definite value for ξ . They only suggest that the previously reported value of ξ should be modified due to the mean field and perhaps off-shell effects.

The final conclusion is that there may not after all exist any definite disagreement between these type of calculations (inspired by the success of Brueckner's many-body theory) and the (in principle) more accurate Monte-Carlo and other methods. The present investigation rather indicates that the basic ideas of Brueckner theory, successful as a theory of nuclei may also be carried over to the unitary system although higher order terms, not customarily included in "Brueckner Theory" would have to be included. The relatively strong correlations expressed by the large effects of mean field insertions and consequently large values of I_w suggest that such higher order terms may be important.

Brueckner type calculations with results similar to the Monte Carlo were also reported by Siu et al[9, 10]. They differ from the present by including also ring diagrams and by the use of different interactions. Although there is some agreement there is at this time not enough ground for a direct comparison of results.

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